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Mathematics News Letter

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Published under auspices of the Louisiana-Mississippi Section of the Mathematical Association of America and the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics.

To mathematics in general, to the following causes in particular is this journal dedicated: (1) the common problems of high school and college mathematics teaching, (2) the disciplines of mathematics, (3) the promotion of M. A. of A. and N. C. of T. of M. projects.

VOL. 3 BATON ROUGE, LA., MARCH, 1929 NO. 7

TEACHERS OF MATHEMATICS! GO TO LAFAYETTE APRIL 12, 13!

The change of dates of the Louisiana-Mississippi Section-Council meeting to APRIL 12, 13, will make it possible for the News Letter to convey this challenge to a large number of the really wide-awake mathematics teachers in our territory.

Five hundred copies of the Letter are printed in this March issue. It will be possible for us to mail out at least four hundred of these to Louisiana-Mississippi mathematicians, high school and college, IF EVERY INDIVIDUAL WHO SHALL RECEIVE THE NEWS LETTER AT ANY TIME BEFORE APRIL FIRST WILL SEND TO EDITOR SANDERS AT ONCE A LIST OF A FEW PEOPLE WHO ARE NOT NOW GETTING IT.

—*S. T. S.*

THE LA.-MISS. BRANCH OF NATIONAL COUNCIL AND MATHEMATICS NEWS LETTER

It should be gratifying to the Louisiana-Mississippi Branch of the National Council to know that our organization was mentioned several times at the national meeting in Cleveland on

February 22-23, and was spoken of as being a vigorous active branch. This should serve as a stimulus to greater achievement. Those who are members should pledge their interest and ability to the cause, and talk the National Council to the mathematics teachers who are not members.

The News Letter came in for a round of praise at Cleveland. Every member of the Louisiana-Mississippi Branch of the Council knows what this journal means to that organization. I have congratulations from Chairman W. C. Roaten on the splendid work of the News Letter.

—*H. S.*

AN APPEAL FOR THE NEWS LETTER

The adoption and successful operation of a scheme of co-operation between the M. A. of A. on the one hand and the Council on the other, has attracted the attention of national leaders in the mathematical field. The Louisiana-Mississippi plan was mentioned with favor at the Nashville meeting of M. A. of A. in December, 1927, and, as shown by President Barber's letter, by the National Council at its recent meeting in Cleveland.

There are, approximately, one thousand high school and college mathematics teachers in Louisiana and Mississippi. IT IS THROUGH THE AGENCY OF SUCH A LOCAL ORGAN AS THE NEWS LETTER THAT EVERY HIGH SCHOOL MATHEMATICS DEPARTMENT IS ABLE TO RENDER OR TO RECEIVE A SERVICE ONCE A MONTH TO OR FROM SOME OTHER DEPARTMENT, COLLEGE OR HIGH SCHOOL, IN THE TWO STATES.

No more powerful argument for the Mathematics News Letter can be advanced than that its pages reflect and promote the joint and correlate interests of secondary and college mathematics and mathematics teaching. It is the circulating organ of a union of the two great classes of mathematical workers, such union having been effected in order to bring about a MASS ADVANCE of all forms of mathematical activity in this part of the South. Among these are to be placed: Increase of professional interest on the part of mathematical workers, a growing sense of the importance of contacts between a mathematics teacher and his fellow workers, increased valuation of

scholarship as a factor in successful teaching, a sharpened vision of the importance of cooperation between high school and college mathematical workers.

Properly used, the Mathematics News Letter can be made the means of bringing to our teachers of mathematics in the home territory the most recent tested-out advances in teaching projects and in mathematical course materials.

—The Editors.

ON THE NATURE OF MATHEMATICAL REASONING

(Second Paper)

In our February issue a partial analysis of the nature of mathematical reasoning was made. The results of the analysis, as far as it was carried, were substantially as follows: Two characteristics, namely, (a), CERTAINTY, (b), ACCURACY, distinguish a mathematical inference. These are consequences of the following conditions.

(a) The meaning of each concept employed in the reasoning is IDENTICAL for all properly informed minds.

(b) The same is true for every operation used.

(c) The meaning of every concept, symbol or operation used is UNIQUE, that is, each concept, symbol, or operation has ONE and only one meaning for the properly informed mind.

(d) The number of facts or concepts of the initial hypothesis is DEFINITE or COUNTABLE.

(e) Distinction between actual truth and hypothetical truth of hypothesis is waived.

(f) Facts (a), (b), (c) contribute a certainty to the inferences made partaking of the nature of the certainty of a perceptive act.

In view of this summary our previous suggestion that mathematics may be defined as the science which makes use only of perfected inferential processes might be again made. An inferential process may be said to have been at least partly perfected when the subject material of the process has been made to conform to the above conditions, namely, (a)—(e).

In illustration of the type of reasoning which is technically regarded as non-mathematical we make use of a passage, taken

practically at random, from Ricardo's Political Economy. The passage is as follows:

"Any improvement in the facility of working the mines, by which the precious metals may be produced with a less quantity of labor will sink the value of money generally. It will then exchange for fewer commodities in all countries; but when any particular country excels in manufactures so as to occasion an infl.ux of money towards it the value of money will be lower, and the prices of corn and labour will be relatively higher in that country than in any other."

Let us re-formulate the content of the paragraph as far as possible after the manner in which a sequence of mathematical inferences is customarily formulated.

(1) If the amount of labor required to mine the precious metals is decreased, then

(2) the value of money is generally decreased, whence

(3) fewer commodities will be exchanged for money.

(4) If a country manufactures more goods than any other country, then

(5) there will be a flow of money towards it, whence

(6) the value of money will be decreased, whence

(7) the values of corn and labor are relatively higher than in other countries.

While granting readily that a properly informed reader of this series of inferences might concede a high degree of probability to the correctness of the inferences, this is far from saying that the deductions have a mathematical character.

On the contrary, the terms, "labour", "value", "money," have not meanings which are, (a), unique, (b), universally accepted to the properly informed. This is shown by the following quotations taken from Palgrave's Dictionary of Political Economy. " 'Adam Smith's Wealth of Nations contained scarcely a definition'—Senior." "McCullough defines labour as including operations of animals, machinery and man." "In general Smith restricts labour to human exertion, but on occasions speaks of laboring cattle." "Says Senior: 'Labour signifies both the act of labouring and the results of that act' ". Criticizing Senior, McCullough says, "Accumulated labour is merely a compendious, though inaccurate, mode of signifying accumulated results of labour" "An exhaustive treatment of such a fundamental

conception as "labour" would involve a discussion of many of the most vexed questions in economics."

The last quotation shows that condition (d), described above as necessary to a mathematical inference, is lacking in Ricardo's inferential process, that is, the facts of the initial hypothesis are not definite, or countable.

Similarily the term "value" has not a universally accepted unique meaning. According to Jevons the "value" of a thing is a function of its utility. In the Karl Marx theory, "value" is a function only of cost of production. The citations are from Palgrave.

Quotations could be adduced showing that some of the functions of "money" are subjects of controversy even among the leaders of economic science, so-called.

Condition (e), necessary to a mathematical process, is not satisfied by the Ricardo reasoning, nor, for that matter, is it possible, in the nature of things, for it to be satisfied by any reasoning that deals with the material of political economy. For, in economics, the distinction between actual and hypothetical truth of hypothesis may not be ignored. On the contrary it is vital in that field.

In concluding this section of our discussion the following remark is in order: According to reports from competent workers in the field of economics there is a strong present tendency to a far more strict mathematical formulation of economic doctrine than there was in the times of Ricardo and Adam Smith.

—*S. T. S.*

The News Letter editorial management is in great need of the names and addresses of many more mathematical workers. Especially are they needed from Mississippi. Increasing the News Letter, subscriptions by circulation and printed appeal from the editors is not easy until this need is supplied. A register of addresses of mathematics teachers for 1927-1928 is not a reliable mailing list for 1928-1929. Too many changes of position have occurred. May we have your help?

Mathematics teachers and scholars representing various sections of America are among those who have places on our contributing staff.

THE LOUISIANA ACADEMY OF SCIENCES

SECOND ANNUAL MEETING

Southwestern Louisiana Institute, Lafayette, La.
Friday and Saturday, April 12 and 13, 1929

The Louisiana Academy of Sciences will hold its Second Annual Meeting at Southwestern Louisiana Institute, Lafayette, La., on Friday and Saturday, April 12 and 13, 1929. The first session will begin on Friday, April 12, at 1:30 p. m.

The coming meeting promises to be better than the last one. The fact that the Louisiana-Mississippi section of the M. A. of A. meets conjointly with the Academy should add greatly to the interest of the meeting. The papers to be presented deal with a variety of subjects and should prove of interest to every one.

Please Make a Special Effort to be Present at Meeting.

Notice: The Constitution requires that notices be sent to the members of the Academy one month in advance of the meeting at which the Constitution is to be amended. The suggestion has been made that the Constitution be amended to the effect that the annual dues be raised form $1.00 to $3.00. This matter will be brought up for consideration at the coming meeting.

A SOCIALIZED LESSON IN ARITHMETIC

By RUTH MARKEY
McDonough No. 12 School, New Orleans, La.

"A marked characteristic of the new century in all progressive nations is the quickened interest in the child and his education." Each school is a center of community life and each pupil can be trained into membership in that community. He can be filled with the spirit of service and provided with the instruments of effective self-direction. The problems there solved reflect the life of the larger society of which the school is a

part. By this a guarantee of a creditable democratic society in the future is assured.

The character of the work performed by a system of schools will be determined by the methods used in the recitation. If we are to make socialized schools, then the methods of instruction employed in the classroom must harmonize with the methods of democracy, which demands citizens capable of independent thought and initiative. An important fact of the school and the recitation period is that there be a presentation of social situations as genuine as those of adult life. Dewey has insisted again and again that "school is life."

In a socialized recitation the emphasis is placed upon pupil participation and cooperation, the teacher keeping in the background as much as in consistent with economy of time. The pupils work together in the gathering of material and in the presentation of it so that the class will be an open forum for discussion. Every pupil who presents a proposition will be obliged to defend it by adequate proof. Instead of waiting for the teacher to ask questions, they will question one another and will carry on the work without direction. When pupils become interested in their work and begin to think for themselves, it is natural for them to ask questions.

There must be a constant effort to make the subject matter of vital interest and of practical value to the pupils by relating it to life and to community interest. "The purpose of a socialized recitation is to do away with passivity in the classroom; to provide opportunity for the natural development of initiative, of activity resulting in originality, of the imaginative powers, and of the realization and asumption of responsibility; to give opportunity for the child to do and to be, rather than merely to know."

In mathematics we find splendid opportunity for an introduction of socialized recitation. The pupil measures his effort with that of his associates, as he must meet their questions and criticisms and prove his conclusions to a group. In a junior business world, he learns to hold his own, to be self-reliant, and to listen to just interruptions and objections. He becomes more original in the new democratic atmosphere. Each pupil knows that he is to be carefully checked by a group of co-workers, who have decided upon a definite reason for certain pro-

gressions in solving given work. Children's work in mathematics ought to be derived from their own needs and their own spirit of inquiry. Each pupil feels a sense of individual responsibility or opportunity in the assignment. He realizes that he is expected to be able to tell the class something new and interesting. "Many pupils have their study habits completely transformed in this way, for there is no keener incentive to careful preparation than the prospect of an interested, respectful appreciative audience."

In no subject has there been so many attempts to make material real, practical and close to life-interest as in mathematics. Teachers need to remember that a problem is not interesting to a pupil unless it is in some way made his own problem.

In the following plan an attempt is made to illustrate how a program of socialized recitations can be utilized in making a general review of the work in mathematics with an Eighth Grade A on the eve of its entrance into high school. Each topic in the mathematics course of study has been considered, with an accompanying example by way of illustration. This plan may take a variety of forms dependent upon the subject-matter, type of lesson, age of pupil, and size of class.

The teacher takes the initial step so that the children will get the proper guidance:

"Children, we are going to make preparations for a field trip through the Vieux Carre' next month in honor of some of our guests for Easter. It will be enjoyable to give a reception afterwards on one of our pleasure boats; we can take a trip on the Mississippi River, through the Industrial Canal and cruise on Lake Pontchartrain. Let our class form an Association of Commerce having a chairman with various committees to make necessary investigations and report their findings to the class. It will be advisable to formulate our plans today. Who shall be our chairman? What committees shall we appoint?"

After the chairman has been selected, the committees will be formed at the suggestion of both the pupils and the teacher. The number and kind will depend upon the nature of the entertainment to be extended and the number of questions invited. Four weeks time will be given to accomplish this undertaking; each committee will be assigned a specific date to give its report.

The following is a suggested list of committees. The chairman will assign the duty and responsibility to each committee:

1. **A reception** committee will determine the number of guests, adequate hotel accommodations, and refreshments needed.

2. The **entertainment** committee will provide the means of affording pleasure. Bids must be gotten on music for dancing and automobiles for sight-seeing.

3. A **boat** committee to make investigations as to the floor space on the boat for dancing, the quantity and quality of decoration, and the kind and amount of incidentals necessary to make it a success.

. An **advertising** staff to show the advantages of "America's Most Interesting City." Prepare graphs indicating the range of temperatures to illustrate our ideal climate. Exports and imports with data as to taxes and duties may also be plotted to prove why New Orleans is the second port in the United States. These graphs may be hung about the boat.

5. An **historical** committee will collect the colorful and unique facts about our quaint old city and its historical land marks in Vieux Carre'.

The following problems illustrate the type of examples that the pupils may offer in each topic.

(1). **Graphs:** Graphs of temperatures; exports and imports, etc.

(2). **Pythagorean Theorem:** (a) Frank was on the decoration committee. He found that the reception hall on the boat is 120 ft. long and 50 ft. wide. What length of ribbon will be required to reach from one corner of the ceiling to the opposite corner?

(b) If one person walks 50 yd. along the Industrial Canal and another crosses the bridge which is 75 ft. wide how far apart will these two persons be?

(3). **Parallel Lines:** Royal and Chartres Streets are perpendicular to Esplanade Avenue. By use of a draftsman's triangle, illustrate the relation between these two streets.

(4). **Concentric and Tangent Circles:** Two games are to be played. In one the circles drawn must be concentric, and in other they must be tangent. Indicate how these fields must be laid off.

(5). **Similar Triangles:** If a tree in Jackson Square 20 ft. high casts a shadow 16 ft., how tall is the Cabildo that casts a shadow 60 ft. long?

(6). **Scale Drawing:** Our new court house occupies a space 350 ft. by 300 ft. Make a charcoal sketch of it using the scale 20 ft. to 1 in.

(7) **Angle Measurements:** (a) If the hands of the Cathedral clock show 12:15 o'clock, what angle is formed at the centre? 12:20 o'clock? 12:30 o'clock? 12:45 o'clock?

(b) Visitors' automobiles are permitted to park at a 45° angle. Show this on paper by means of a protractor.

(8) **Rectangle** The band for dancing is seated on a platform 10 ft. by 8 ft. wide. How many sq. yd. does it occupy?

(9) **Triangle:** A triangular garden in one of our patios, or courtyards, in the French Quarters is 6 ft. long and 1 yd. across. How much space is there to plant some flowers?

(10) **Trapezoid:** One of our northern visitors saw an attractive looking spot facing Esplanade Ave. It was a trapezoid 80 yd. long in the rear and 60 yd. long across the front and 100 yd. deep. How much will it cost him to buy it at $3000 per acre?

(11) **Prison:** Little cotton bales 5 in., by 3 in. by 2 in. were given as favors. How many favors can be made from 20 cu. ft. of cotton?

(12) **Cylinder:** How many gallons of punch can be put in a cylindrical bowl 14 in. deep and 2 ft. in diameter?

(13) **Cone:** How many ice-cream cones 2 in. in diameter and 4 in. deep can be filled from a freezer 8in. in diameter and $2\frac{1}{2}$ ft. deep?

(14). **Pyramid:** The steeples of the St. Louis Cathedral are pyramids. If we were to represent them by smaller ones, 3 ft. by 2 ft. by 10 ft., how many cu. ft. would each model contain?

(15) **Sphere:** Outside of the Louisiana Historical Museum are found a number of machine gun balls 8 in. in diameter. What is the surface of each one?

The method of handling each lesson may be determined by the group responsible for that day's report. It may be done as seat work or at the board; it may be effectively carried out by having contests, games, or trying to beat one's own

record. The number of problems in a day's lesson will vary with the nature of the subject-matter and the ability of the class.

This makes evident the fact that a socialized recitation is the outcome of practical experiments to create an atmosphere of activity and responsibility for the child in the classroom. It lays stress upon self-control and activity through experiences created in the classroom for the purpose of training the child by having him cooperate with his schoolmates in some essential and profitable work. It makes the school-life real and natural, and does not neglect any fundamental principle of good teaching. When effective social accomplishment of some kind is the aim placed before pupils, it inspires them to better efforts and prmits the teacher to act as their helper and adviser in the great art of living.

"Any exercise which tends to further social activity or social outlook, which gives insight into social conditions and usages or which influences the attitude toward society, may be regarded as a socializing exercise."

GREETINGS

Exeter, N. H., March 3, 1929.

My dear Mr. Schroeder:

This is to bring you greetings from the National Council and to let you know that the Louisiana-Mississippi Branch was mentioned several times at our annual meeting in Cleveland.

You were referred to as a vigorous, active branch of which the Council is very proud. The News Letter was commended as a particularly fine piece of work and held up as a model for others.

Your members may be glad to hear of this appreciation on the part of the parent organization.

Please find inclosed $1.00 for my subscription to the News Letter. I hope that you will feel free to call on us at any time when we can help you.

Cordially,

HARRY C. BARBER, President

MATHEMATICS USED TO EXPLAIN PHYSICO-CHEMICAL PHENOMENA

By J. H. COLVIN
Louisiana State University

"If one is sufficiently lavish with time, everything possible happens."—Herodotus.

The introduction of the conception of the discontinuity of matter makes us think of all matter being composed of millions of particles instead of being in a continous state. Due to this fact physicists, after being faced with the impossibility of applying the laws of rational mechanics to such a very complex phenomenon, have had to make use of more mathematics—the calculus of probabilities.

The calculus of probabilities has therefore the inestimable advantage that it allows us to pursue our investigation into a domain where the methods of rational mechanics are powerless. But this advantage is compensated somewhat by serious disadvantages chief of which is the ease of making a false step. The uncertainties which the calculus of probabilities introduces cannot be better demonstrated than by recalling the very definition of probability.

The probability of an event is the ratio of the number of cases favorable to the event to the total number of possible cases, all the possible cases being considered to be equally probable. It is the last reservation, necessary though it is neglected.

So, finally, the agreement of the consequences of the hypothesis with experience is the sole criterion which we can invoke in its favor.

If we let 1 *cc* of any gas communicate with 1 *cc of* any other gas at 0° C and 760 *mm* pressure a mixture of the two gases will be the resultant there being about $4: 10^{19}$ molecules present. The most accurate analysis of a sample from any portion of the volume will show 50% of each gas. No matter how long we should wait analysis would show the same percentage composition. These facts would lead us to conclude that any portion of the volume would always contain 50% of each gas. But is it true? It is only true so far as our observation goes. We would ordinarily never expect the two gases

to separate, one gas occupying one half of the vessel and the other gas occupying the other half. This, then, is what we would call an absolutely irreversible phenomenon. But is it absolutely irreversible?

The calculus of probabilities shows us that there is a chance that the two gases will return to the original state. That chance is 1: ${}_{4\cdot 10^{19}}P_{4\cdot 10^{19}}$ which is a relation we are unable to appreciate. But even though the chance is so extremely small the chance is still there and makes some of our physico-chemical laws less absolute, and instead of thinking of them in terms of an "absolute determinism" we must think of them as being a "statistical determinism."

Statistical laws, which for a long time seemed to apply exclusively to the biological, social, economic, and kindred sciences-precisely on account of the extreme complexity of the phenomena in these sciences, and because of the impossibility, generally experienced, of discerning the causes which produced them and made them vary—have been extended, little by little, by means of the calculus of probabilities to what are usually termed the "exact sciences." It seems as though these latter sciences, and particularly physical chemistry, only, owe their title of exact sciences to the law of the large numbers, which usually renders the effects of fluctuations inappreciable.

In recent years these statistical laws have been introduced with particular intensity into physical chemistry, following the conception of the granular structure of matter and the generalization of the kinetic theories. Thus even the study of fluctuations and of their consequences has given to these theoretical conceptions a reality which may be termed experimental.

The tendency is to replace the physico-chemical law, which we have been accustomed to regard as final and inevitable, by the statistical law, which, theoretically at least, is liable to very rare exceptions.

Thus this new conception tends to replace the absolute determinism of the laws of physics and chemistry, as we observe them, by a kind of larger statistical dterminism.

The keen interest with which, in the same field, widely distant groups of workers view one another's programs is one of the characteristics of scientific development.

PROBLEMS FROM THE FIELD OF INVESTMENT

By IRBY C. NICHOLS
Louisiana State University

From time to time, requests of solutions of certain problems come from investors in the business world. Below are given the statements and solutions of a few of these which seem to be typical:

Problem I. Finance Company A loans Citizen B \$100.00. *A retains* \$8.00 *as interest paid in advance. B gives A his personal noté for* \$100.00, *payable in* 10 *equal monthly installments, the first to be paid one month from date of his note. If there be no further charges, what rate of interest does B pay A?*

Solution. The present value of A's loan to B is \$100.00 less \$8.00 or \$92.00. Since the amount K of any sum of money whose present value is P, put at interest i, and compounded for n years, is $P(1+i)^n$ we have the fundamental relation $K=P(1+i)^n$ whence $P=K/(1+i)^n$. Similarly the present value of B's first montly payment of \$10 due one month hence is $10/(1+i)^{1/12}$ where i is the rate of interest to be found, and 1/12 is the time, one month, expressed in years. Similarly for B's installments due in 2, 3, 4, --- 10 months respectively from the date of his note. Their respective present values are $10/(1+i)^{2/12}$, $10/(1+i)^{3/12}$, --- $10/(1+i)^{10/12}$. The sum of the present values of these installments gives the total present value of B's payments to A, which amount should equal to present value of A's loan to B. Hence the equation $92.00=10/(1+i)^{1/12}+10/(1+i)^{2/12}$ $10/(1+i)^{3/12}+ ---- +10/(1+i)^{10/12}$.

Factoring out 10 and using negative exponents, we have $9.2=(1+i)^{-1/12}+(1+i)^{-2/12}+ --- +(1+i)^{-10/12}$. Our right member is a geometrical series whose first term a is $(1+i)^{-1/12}$, whose ratio r is $(1+i)^{-1/12}$, whose term l is $(1+i)^{-10/12}$ and whose number of terms n is 10. The formula for the sum S of n terms of a geometrical series is $(a—rl)/(1—r)$. Applying this formula here we have

$$9.2=\frac{(1+i)^{-1/12}-(1+i)^{-1/12}(1+i)^{-10/12}}{1-(1+i)^{-1/12}}$$

Whence we get the equation

$9.2(1+i)^{11/12}-10.2\ (1+i)^{10/12}+1=0$, solving which equation for i, we obtain $i=20.2\%$; that is B pays A 20.02 per cent interest.*

Problem II. A loan company A lends Citizen B \$100.00, *but retains* 5.8 *per cent or* \$5.80 *as prepaid interest and* \$3.00 *as a service charge for investigating B's financial risk and in preparing and recording necessary legal papers covering the loan. B repays the loan in* 10 *monthly payments of* \$10.00 *each, the first payment being made one month after the date of the loan. What rate of interest does B pay A?*

Solution. The solution of this problem follows closely that of problem above. Here the present, or cash value of A's loan to B is \$91.20. Again the total cash or present value of B's monthly installments is $10/(1+i)^{1/12}+10/(1+i)^{2/12}+10/(1+i)^{3/12}$ $----+10/(1+i)^{10/12}$. Whence, as above,

$9.12(1+i)^{11/12}-10.12(1+i)^{10/12}+1=0$. Solving this equation for i, we find $i=22.6\%$, the rate of interest which B pays A.

Problem III. In purchasing a home, Citizen B secures the assistance of a loan through a building and loan association of his community. For each \$100.00 *of the loan, B repays monthly* \$1.00, *the first repayment being made January* 1, 1929, *the date of the loan. Of each* \$1.00 *repaid by B, the association retains* 65 *cents as interest due on the loan, and credits the remaining* 35 *cents on B's principal. On all sums credited to his principal, the association allows B* 7 *per cent interest, compounded semi-annually, January* 1 *and July* 1, *of each year.* (*a*) *How long will it take B to repay his loan?* (*b*) *What rate of interest does B receive on his accumulations?* (*c*) *What rate of interest does B pay the association if he pays no extra charges?* (*d*) *What rate of interest does he pay the association if he is required to pay* \$52.50 *as service charges in securing a loan of* \$4500.00?

Solution. (a) Let n equal the number of years required of B in which to repay his loan. Then $2n$ will equal the number of six months periods in n years. On July 1, 1929, B will receive a credit on his principal of six times 35 cents plus interest on 35 cents for 6, 5, 4, 3, 2, and 1 months respectively, or a total of $\$0.35(24.49/4)=\$2.142875$. This sum will bear 7 per

*[In this problem and also in others of this paper, Glover's seven-place table of logarithms is used.]

cent interest, compounded semi-annually for $2n-1$ periods. Similarly for the second six months, the accumulated amount of it for the remaining life of the loan is \$2.14287 $(1.035)^{2n-2}$ and so on for each six months of the n years of the life of the loan, or until a total of \$100.00 has been accumulated by B with which to cancel his debt. Hence we have

$$2.142875(1.035)^{2n-1}+2.142875(1.035)^{2n-2}+\cdots+2.142875(1.035)^{1}+2.142875=100.$$

Factoring out 2.142875 we may write,

$$2.142875(1.035^{2n-1}+1.035^{2n-2}+\cdots+1)=100.$$

The expression in parentheses is a geometrical series, whose sum is $(1.035^{2n}-1)/.035$.

Hence we have $2.142875(1.035^{2n}-1)/.035=100$. Reducing and solving for 1.035^{2n}, will have

$1.035^{2n}=(64.49/24.49)$. Whence, by logarithms,

$$n=\frac{\log 64.49-\log 24.49}{2\log 1.035}=14.073.$$

Therefore it will take B 14.073 years or 14 years and 6.6 days to repay his loan.

(b) In general, if i be the effective rate of interest, j the nominal rate, and m the number of times an amount is converted annually, we have the fundamental relation $1+i=(1+j/_m)^m$, whence $i=(1+j/_m)^m-1$. Applying this relation to our present question, we have $i=(1+.07/2)^2-1=(1.035)^2-1=.071225$; that is, the effective rate which B receives is 7.1225 per cent. This is nearly $7^1/_8$ per cent.

(c) B pays \$0.65 per month, in advance, as interest due on a \$100.00, or \$0.0065 per month per dollar. Since it is paid in advance, B really receives the use of \$0.9935 instead of a whole dollar. So \$0.9935 pays a rental per month of \$0.0065. Using the standard relation given in (b) above,

$$j/_m=\frac{12(.0065/.9935)}{12}=.0065/.9935. \quad \text{Whence } 1+i=(1+j/_m)^m$$

$=(1+.0065/.9935)^{12}=1.0814$. Therefore $i=.0814$ or 8.14 per cent interest.

(d) If B pays an extra charge of \$52.50 to secure a loan of \$4,500.00, he must pay enough interest on \$4,447.50, the amount of cash actually received by him, to equal the volume of

revenue expected by his association from \$4,500.00. Hence $i=(4500.00/4447.50)/.0814=.08236$ or 8.236 per cent; that is, B pays 8.236 per cent on the money received from his association.

Problem IV. If A be in the small loan business and keeps no idle money on hand, what per cent interest does he realize if he loans his money at $3^1/^2$ per cent per month?

Solution. A's turnover is 12 times per year. From the fundamental relation $1+i=(1+j/_m)^m$, given above, we have $1+i=(1+.035)^{12}$, since $j/_m$ is the periodic (here monthly) rate .035 and m is 12, the number of conversion periods per year. Hence $i=(1.035)^{12}-1=1.5110687-1=.5110687$, or about 51.1 per cent.

What annual payment will be required of A to liquidate a debt to B of \$1000.00, *together with* 7% *interest, in* 3 *equal annual payments?*

Solution: A problem of this nature appears simpler if it be considered as two transactions, operating separtely, until the time of final settlement, which here is 3 years hence. It does not matter whether A's annual payments be credited directly on a debt to B, or placed with some third party C to draw interest at 7% until the end of 3 years. Treated thus, at the end of 3 years B would expect from A an amount $S_1=1000(1.07)^3$. A would have on hand with C an amount S_2, the sum of his annual payments compounded at 7% for 2, 1, and 0 years respectively. By hypothesis, each year A has placed with C the same amount, say P dollars. Then

$$S_2=P(1.07)^2+P(1.07)^1+P$$
$$=P(1.07^2+1.07+1)=P(3.2149).$$

Now since debt S_1 equals amount on hand S_2 we have $P(3.2149)=1000(1.07)^3$. Whence solving for P we have

$$P=1000(1.07)^3/3.2149=381.05$$

Therefore A's annual payment to cancel a debt of \$1000.00 together with 7% interest on same in 3 years is \$381.05.

Readers of the News Letter will be interested in being informed that in the not-too-distant future a series of articles may be published showing the results of a thorough investigation of the disciplinary values of mathematical study. Miss Vevia Blair, of the Horace Mann's School for girls will be the writer of the series.

AN OBSERVATION ABOUT STATISTICAL FORMULAE

By W. PAUL WEBBER
Louisiana State University

It may not have occurred to some who use statistical formulas that the most elementary formulas of statistical method have their parallels in the so called exact sciences or have been directly transferred from pure mathematics to perform service in the social sciences and in business. Some examples will illustrate.

(a) The formula for the **mean** of a set of numbers $a_1, a_2, ---, a_n$ with frequencies $w_1, w_2, --- w_n$, viz.,

$$M = \frac{a_1 w_1 + a_2 w_2 + --- a_n w_n}{w_1 + w_2 + --- + w}$$

found in all books on statistics, serves equally well to find the centroid or center of gravity af a set of a masses $w_1, w_2 ---, w_n$ arranged on a straight line at distances $a_1, a_1, ---, a_n$ from some fixed point on the line. This formula is found in every text on mechanics and physics.

When matter is continuously distributed on the line the formula takes the form of an integral as

$$M = \frac{\int_a^b x\, dw}{\int_a^b dw}$$

One might say that the mean grade for a class in arithmetic or in English is the center of gravity of the grades of the individuals arranged along a straight line at distances from a fixed point point in order of magnitdue of the grades.

This shows that the problems in activities apparently far remote from one another are merely special cases of a general principle expressed in mathematical form in symbols without concrete content and equally available to all.

(a) The *standard deviation* of a set of numbers whose individual deviations from some fixed value are $x_1, x_2, ---, x_n$ and frequencies $w_1, w_2, ---, w_n$ is

$$d = \sqrt{\frac{w_1 x_1 + w_2 x_2^2 + --- + w_n x^2_n}{w_1 + w_2 + --- + w_n}}$$

This is precisely the radius of gyration of a set of masses $w_1, w_2, ---, w_n$ at distances $x_1, x_2, ---, x_n$ from the center of rotation. For any one who is familiar with rotating wheels and with the effect of making the rim of a wheel heavy, it easy to see why this formula was selected to measure dispersion.

In the case of a continuous distribution along a line the above formula passes over into an integral as

$$Q=\sqrt{\frac{\int_a^b x\,dw}{\int_b^a dw}}$$

where dw is an infinitesinral element of mass and x its distance from the axis or point of reference.

(c) Consider Pearson's coefficient of correlation,

$$C=\frac{n}{1}\left\{\frac{x_1y_1+x_2y_2+ --- +x_n y_n}{d_1d_2}\right\}$$

It is easy to show that this is precisely the cosine of the angle between two lines having direction cosines x_1, x_2 ---, and x_n and $y, y_2, ---, y_n$ in n-dimensional space, respectively. Now if this cosine is 1, the lines are parallel and the correlation is perfect. If the cosine is zero, the lines are perpendicular and there is no correlation. If the cosine is negative, the lines make an obtuse angle with each other and the correlation is negative or inverse.

NOTE ON THE QUADRATIC FORMULA

By H. L. SMITH
Louisiana State University

The reader well knows that the solution of the equation

$$(1)\qquad ax^2+bx+c=0$$

is given by the formulas

$$(2)\qquad x=\frac{-b+\sqrt{b^2-4ac}}{2a},\quad \frac{-b-\sqrt{b^2-4ac}}{2a}$$

if a is not zero. If a is zero but b is not, the equation (1) still has the solution $x=-c/b$; but this solution is not given by (2). It is natural to ask if it is possible to put (2) into such a form that they give the solution of (1) not only in the case in which

a is not zero but also in the case in which a is zero but b is not. We shall show that this can indeed be done.

To this end let r be either square root of $b^2—4ac$ is not zero but let it be $—b$ if ac is zero. Then, whether or not ac is zero, r will be $\sqrt{b^2—4ac}$ or $—\sqrt{b^2—4ac}$, and (2) may be written

$$(3) \qquad x = \frac{-b+r}{2a}, \quad \frac{-b-r}{2a}$$

But

$$(-b+r)\,(-b-r) = b^2 - r^2 = b^2 - (b^2 - 4ac) = 4ac$$

so that

$$\frac{-b-r}{2a} = \frac{2c}{-b+r}$$

Hence (3), and therefore (2), may be written

$$(4) \qquad x = \frac{r-b}{2a}, \quad \frac{2c}{r-b}.$$

The formulas (4) have the desired properties as the reader will easily verify.

PROBLEM OF TANGENCY

By R. L. O'QUINN
Louisiana State University

Problem. Given two points $P_1\ (x_1\ y_1)$, $P_2(x_2\ y_2)$ and a line l: $ax+by+c=0$, to find the center of the circle through P_1 and P_2 tangent to the line l.

Let Q (u, v) be the required center. Then Q is on the line

$$\begin{cases} x = \frac{1}{2}(x_1+x_2) + \frac{1}{2}(y_1-y_2)t \\ y = \frac{1}{2}(y_1+y_2) - \frac{1}{2}(x_1-x_2)t; \end{cases}$$

that is, there is a t_0 such that

$$(1) \qquad \begin{aligned} u &= \tfrac{1}{2}(x_1+x_2) + \tfrac{1}{2}Yt_0 \\ v &= \tfrac{1}{2}(y_1+y_2) - \tfrac{1}{2}X\,t_0, \text{ where } X = x_1 - x_2,\ Y = y_1 - y_2. \end{aligned}$$

We have also

(2) $r^2[(x_1-u)^2+(y_1-v)^2]=(au+bv+c)^2$, where $r^2=a^2+b^2$;

and the problem has a solution if and only if (2) has a solution for t_0 when (1) is substituted into (2). But, by (1), we have

(3) $x_1-u=\frac{1}{2}(X-Yt_0)$, $y_1-v=\frac{1}{2}(Y+Xt_0)$.

Moreover, if we set

(4) $D_1=ax_1+by_1+c$, $D_2=ax_2+by_2+c$, $D=D_1+D_2$, $E=aY-bX$, we have

(5) $au+bv+c=\frac{1}{2}(D+Et_0)$.

Thus (2) becomes

$$r^2[(X-Yt_0)^2+(Y+Xt_0)^2]=(D+Et_0)^2,$$

or

(6) $[r^2(X^2+Y^2)-E^2]t_0{}^2-2DEt_0+[r^2(X^2+Y^2)-D^2]=0$. Here

Here

$$r^2(X^2+Y^2)-E^2=r^2(X^2+Y^2)-(a^2Y^2-2abXY+b^2X^2)$$

$$=b^2Y^2+2abXY+a^2X^2=(aX+bY)^2=(D_1-D_2)^2;$$ that is,

results

(7) $r^2(X^2+Y^2)-E^2=(D_1-D_2)^2$.

The discussion breaks up into several cases.

Case I. $D_1=D_2$. P_1 and P_2 are equidistant from l and on the same side of l.

In this case (6) reduces to

(6^1) $2DEt_0=r^2(X^2+Y^2)-D^2$.

(7) $\begin{vmatrix} a^2+b^2 & aX+bY \\ aX+bY & X^2+Y^2 \end{vmatrix} = \begin{vmatrix} a & b \\ X & Y \end{vmatrix}^2 = D^2$; also,

(8) $(a^2+b^2)(X^2+Y^2)-(aX+bY)^2=(aY-bX)^2=E^2$. Hence

(9) $(a^2+b^2)(X^2+Y^2)=(aY-bX)^2=E$, since $aX+bY=D_1-D_2$

Hence $E_2 \geq 0$, and E not equal to 0.

(A) $D=0$. (P_1, P_2 both on l.) Then (6^1) becomes

$0=r^2(X^2+Y^2)$ not equal to 0. There is no solution.

(B) D not equal to 0. (Neither P_1 nor P_2 is on l. In this case (6^1) has the unique solution

(10) $t_0=\dfrac{r^2(X^2+Y^2)-D^2}{2DE}=\dfrac{E}{2D}$. There is then one

solution given by (10) and (1).

Case II. D_1 not equal to D_2. P_1 and P_2 are on opposite sides of l, or they are on the same side of l at unequal distances from l. In this case (6) is a quadratic equation.

The discriminat $\triangle$ is given by

(11) $\triangle=D^2E^2-[r^2(X^2+Y^2)-E^2]\ [r^2(X^2+Y^2)-D^2]$

$=r^2(X^2+Y^2)[D^2+E^2-r^2(X^2+Y^2)]$

$=r^2(X^2+Y^2)[(D_1+D_2)^2-(D_1-D_2)^2]$ (by (4)), or

(12) $\triangle=4r^2(X^2+Y^2)\ D_1\ D_2$. Hence the solution of (6) is

$$\frac{DE \pm 2\sqrt{[r^2(E^2+Y^2)\ D_1\ D_2]}}{D_1-D_2)^2}, \text{ or}$$

$$(13)\quad t_0 = \frac{(D_1+D_2)(aY-bX) \pm 2\sqrt{[(a^2+b^2)(X^2+Y^2)\ D_1\ D_2.]}}{(D_1-D_2)^2.}$$

or

Case A. $D_1 \cdot D_2 < 0$. P_1 and P_2 are on opposite sides of l. There is no solution.

Case B. $D_1 \cdot D_2 = 0$. One of the points is on l. There is one solution.

Case C. $D_1 \cdot D_2 > 0$. P_1 and P_2 are both on the same side of l, unequally distant from l. There are two solutions. In each case the solutions are given by (13) in (1).

PROBLEMS

Proposed by H. L. SMITH

1. Show that in any triangle

$$\frac{S^2}{K} = \frac{\tan \tfrac{1}{2}(A+B)}{\tan \tfrac{1}{2}A \tan \tfrac{1}{2}B} = \operatorname{ctn} \tfrac{1}{2}A \operatorname{ctn} \tfrac{1}{2}B \operatorname{ctn} \tfrac{1}{2}C.$$

where A, B, C are the angles of the triangle, S is its semiperimeter, and K is its area.

2. Show that in any triangle

$$\frac{S^2}{K} \leqq \frac{(1+\sqrt{1-e^2})}{e\sqrt{1-e^2}} < \frac{8}{e},$$

where s, K are as above, and e is a positive number, at most equal to the smallest of the sines of the angles of the triangle.

Proposed by H. L. Smith: Show that in any triangle $\frac{K}{S^2} = \frac{\tan \frac{1}{2} A \tan \frac{1}{2} B}{\tan \frac{1}{2}(A+B)}$, where A, B are any two angles of the triangle, S its semiperimeter, and K its area.

Solved by S. T. Sanders: It is easily shown that $\tan \frac{1}{2} A = \frac{1-\cos A}{\sin A}$. Applying this relation to each of the three functions

of the right member of the equality to be established, writing sin A, sin B, sin (A+B)=sin C, as $\sqrt{1-\cos^2 A}$, $\sqrt{1-\cos^2 B}$, $\sqrt{1-\cos^2 C}$, respectively, and reducing, we have

$$(1)\qquad \frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}A \tan \frac{1}{2}B} = \frac{\sqrt{1+\cos A}\,\sqrt{1+\cos B}\,\sqrt{1+\cos C}}{\sqrt{1-\cos A}\,\sqrt{1-\cos B}\,\sqrt{1-\cos C}}$$

By the Law of Cosines, cos A, cos B, cos C, are respectively equal to $\frac{b^2+c^2-a^2}{2bc}$, $\frac{a^2+c^2-b^2}{2ac}$, $\frac{a^2+b^2-c^2}{2ab}$.

Placing these values of cos A, cos B, cos C in the right member of (1), simplifying under the radicals, and expressing the radicands in terms of S, reducing, we have

$$(2)\qquad \frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}A \tan \frac{1}{2}B} = \frac{\sqrt{s(s-a)}\,\sqrt{s(s-b)}\,\sqrt{s(s-c)}}{\sqrt{(s-b)(s-c}\,\sqrt{(s-a)(s-c)}\,\sqrt{(s-a)(s-b)}}$$

$$= \frac{s\sqrt{s}}{\sqrt{(s-a)(s-b)(s-c)}}$$

$$= \frac{s^2}{\sqrt{s(s-a)(s-b)(s-c)}}$$

or, $$\frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}A \tan \frac{1}{2}B} = \frac{s^2}{K},\quad q.\ e.\ d.$$

The News Letter is to be congratulated upon having in prospect the following contributors. These are but a few of those who have agreed to contribute regularly to its pages:

Professor H. E. Slaught, University of Chicago, "M. A. of A. Projects and Programs" and "Review of Important Articles in the Monthly."

Professor W. D. Reeve, Columbia University, "National Council Programs and Projects."

Professor Harry Gwinner, University of Maryland, "Sketches of Successful Mathematicians" and "Mathematics as a Tool."

Dean J. A. Hardin, Centenary College, "Sectioning Classes in Mathematics According to Different Abilities."

MAKE YOUR PLANS TO BE THERE!

Friday, March 16, 1929

Exactly four weeks from today i. e., on Friday, April 12, the Louisiana-Mississippi Section of the Mathematical Association of America will meet at Lafayette, Louisiana. The time is not too far off nor too near, it is just right to plan to be there.

We shall meet this year as usual with the National Council of Mathematics Teachers. In addition, we shall have with us at that time the Louisiana Academy of Sciences. These three societies should make a very interesting triangular program.

Friday morning of April 12 will be employed with registration and social activities. In the afternoon the High School division of our program will take place. At this time, also, the Louisiana Academy of Sciences will have its second meeting.

Our the evening of Friday we will have our usual dinner and program. I am not able at this time to state what our "feature" on this program will be. The change of date from March 28 required a change in this part of our program, but we shall have some visitor of note.

On the morning of Saturday, April 13 we shall share the program with the Louisiana Academy of Sciences, the sessions closing with a business meeting and election of officers.

Make your plans NOW to be there.

Respectfully yours,

B. E. MITCHELL,
Chairman La.-Miss. Sec. M. A. A.